Calculations for Broué's abelian defect group conjecture
ブルエの可換不足群予想の計算

千葉大学 理学部 越谷 重夫 (Shigeo Koshitani)
Department of Mathematics and Informatics,
Faculty of Science, Chiba University
e-mail koshitan@math.s.chiba-u.ac.jp

This is a joint work with **Naoko Kunugi** and **Katsushi Waki**, and a detailed version of a result presented here is in [6].

It has been conjectured by Michel Broué that a block algebra of a finite group should be derived (Rickard) equivalent to a block algebra of the normalizer of a common defect group which correspond each other via the Brauer correspondence provided the defect group is abelian, see [2, 6.2.Question]. This is known as *Broué's Abelian Defect Group Conjecture*, (ADGC) for short. We have been continuing a project on Broué's ADGC for a specific defect group, say the elementary abelian group of order nine, see [3], [4], [5]. Our main result here is the following:

Theorem (Koshitani-Kunugi-Waki, 2005)**.** *Let G be the Janko simple group J_4, and let $(\mathcal{O}, \mathcal{K}, k)$ be a splitting 3-modular system for all subgroups of G, namely, $\mathcal{O}$ is a complete discrete valuation ring of rank one such that $\mathcal{K}$ is the quotient field of $\mathcal{O}$ with $\mathrm{char}(\mathcal{K}) = 0$ and such that k is the residue field of $\mathcal{O}$, namely $k = \mathcal{O}/\mathrm{rad}(\mathcal{O})$, with $\mathrm{char}(k) = 3$, and $\mathcal{K}$ and k are both splitting fields for all subgroups of G. Let A be a unique block algebra of $\mathcal{O}G$ whose defect group P is elementary abelian of order 9, and let B be the Brauer correspondent of A in $\mathcal{O}H$ where $H = N_G(P)$. Then, A and B are derived (Rickard) equivalent. In fact, even stronger fact is proved, namely, A and B are splendidly derived (Rickard) equivalent, see* [9] *and* [10].

Remark. In our proof results in papers of Okuyama [7] and [8] are important.

Corollary. *It turns out that Broué's ADGC holds for any prime p and any block algebra of G. This means that Broué's ADGC is settled for all primes and all block algebras of J_4.*

Proof. This follows immediately from Theorem and [1, Lemma 5.1].

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