On bell-shape property of distributions in a subclass of infinitely divisible distributions on $\mathbb{R}$ and applications

Makoto Yamazato
Tokyo Woman's Christian University

A probability density f is called bell shaped if the number of zero points of the n-th derivative of f is n for each $n \geq 0$. It is well known that normal distributions have this property. Concept of bell-shape is deeply connected with total positivity [2]. Gawronski [1] asserted that all stable distributions on $\mathbb{R}$ are bell shaped. However T. Simon [3] pointed out an assertion which was used in Gawronski's proof is incorrect. So Gawronski's assertion was not known to be correct. In [3], Simon proved that positive stable densities are bell-shaped using Yamazato's decomposition of stable densities which is used in [4] for the simple proof of unimodality of stable distributions.

In this talk, extending Simon's method, we prove the bell-shape of distributions in a class of infinitely divisible distributions on $\mathbb{R}$ and as an application we show that all stable distributions on $\mathbb{R}$ are bell-shaped. Furthermore, we point out that the hitting time distribution of 1-dimensional generalized diffusion process is bell-shaped as far as the support of its speed measure contains infinitely many points between starting point and hitting point. This fact is easily seen by the result in [5] combined with the result in this talk.

References

[1] W. Gawronski, On the bell-shape of stable densities, Ann. Probab. 12 (1984), 230-242.

[2] S. Karlin, "*Total Positivity*", Vol. 1, Stanford Univ. (1968), Stanford.

[3] T. Simon, Positive stable densities and the bell-shape, preprint

[4] M. Yamazato, On strongly unimodal infinitely divisible distributions, Ann. Probab. 10 (1982), 589-601.

[5] M. Yamazato, Characterization of the class of hitting time distributions of 1-dimensional generalized diffusion processes, Proc. 6th Japan-USSR Symposium on Probability Theory and Mathematical Statistics, (1992) 422-428, World Scientific, Singapore-New Jersey-London-Hong Kong.